$$\int_o^H \epsilon_S dh = \frac{M_S}{\rho_S A} \quad (7)$$

If the solid holdup is not a function of height in the bed, Equation (7) reduces to Equation (3).

The results shown in Figure 3 illustrate the shortcomings of the distinct bed height and the uniform bed assumptions used by previous investigators. In this case, which is typical of each solid used in both columns, the solid holdup slowly drops to zero from about 30 to 70 cm. This transition region is a significant fraction of the total bed height, and it must be considered in the realistic design of three-phase systems.

## CONCLUSIONS

The electroconductivity technique described here can be used not only for determining the overall phase holdups in a three-phase fluidized bed, but, more importantly, it can also be used for determining the local holdups as a function of height in the column.

One disadvantage of the technique is that it can only be applied to systems with electroconductive liquids. However, since most real or prototype systems use either water or can be simulated with a fluid that can readily be made electroconductive, this handicap does not seem to be too severe.

The technique has been applied successfully to a number of systems, including porous alumina beads, if a correction is made for their internal porosity. It has shown the existence of the transition region as the bed goes from a three-phase to a two-phase system. Further work should result in correlations for the distribution of the three phases throughout the entire column. These predictive equations will help in the rational design of reactors in which local conditions throughout the bed must be considered.


## ACKNOWLEDGMENT

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## NOTATION

$A$ = cross-sectional area of the bed
$g$ = gravitational acceleration
$h$ = position down the column
$H$ = expanded bed height
$M_S$ = mass of solids
$P$ = local pressure in column
$\gamma$ = conductivity in the bed
$\gamma_o$ = conductivity in liquid alone
$\epsilon$ = phase holdup
$\rho$ = phase density

### Subscripts

$G$ = gas phase
$L$ = liquid phase
$S$ = solid phase

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# Application of a Self-Consistent Model to the Permeability of a Fixed Swarm of Permeable Spheres


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Cell models have been widely used for calculating the properties of concentrated suspensions, particularly the permeability of a porous medium composed of spherical particles. A cell consists of a particle surrounded by fluid; the models differ in the conditions imposed on the outer boundary of the cell (a full description is given


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by Happel and Brenner, 1965). The free cell model of Happel (1958) is the most quoted. Recently, Neale and Nader (1974) successfully used a self-consistent scheme for calculating the permeability of a porous medium; briefly, the cell is immersed in the porous medium whose permeability is the unknown quantity. These two models have been used for calculating the permeability of a porous medium composed of impermeable spheres.

To our knowledge, only the free cell model has been applied to a porous medium consisting of permeable spheres (Neale et al., 1973), and the influence of the aggregates on the permeability of the whole medium is clearly shown. Moreover, the theoretical predictions based upon Brinkman's extension of Darcy's law for a single isolated permeable sphere were recently experimentally confirmed (Matsumoto and Suganuma, 1977).

The purpose of this paper is the calculation of the permeability of a fixed swarm of permeable spheres, using the model of Neale and Nader (1974). The mathematical problem is briefly exposed, and results are discussed and compared with the free cell model.

## MATHEMATICAL DESCRIPTION OF THE SYSTEM

### Equations and Boundary Conditions

The following set of familiar equations is written for the three zones of the model (Figure 1):

$$r \leq R: \quad \nabla \hat{p} = \mu \nabla^2 \hat{\mathbf{u}} - \frac{\mu}{k_1} \cdot \hat{\mathbf{u}}; \quad \nabla \cdot \hat{\mathbf{u}} = 0 \qquad (1a)$$

$$R \leq r \leq S: \quad \nabla p = \mu \nabla^2 \mathbf{u}; \quad \nabla \cdot \mathbf{u} = 0 \qquad (1b)$$

$$S \leq r: \quad \nabla \tilde{p} = \mu \nabla^2 \tilde{\mathbf{u}} - \frac{\mu}{k} \cdot \tilde{\mathbf{u}}; \quad \nabla \cdot \tilde{\mathbf{u}} = 0 \qquad (1c)$$

The permeability $k_1$ of the permeable sphere is supposed to be given, while the permeability $k$ of the whole medium is the unknown of the problem.

Continuity of pressure, velocity, and tangential stress is imposed at $r = R$ and S. Velocities are finite at $r = 0$; at $r = \infty$ velocity is equal to the constant mainstream velocity $U$ (Figure 1).

### Solution

The problem is solved using stream functions in spherical coordinates. According to the symmetry of the problem, they were supposed to be proportional to $\sin^2\theta$. Thus, general solutions for the set of Equations (1) can be expressed as (Happel and Brenner, 1965; Neale et al., 1973)

$$\hat{\psi} = -\frac{k_1 U}{2} \cdot \left[ \frac{A}{\xi} + B\xi^2 + C\left(\frac{ch\xi}{\xi} - sh\xi\right) + D \cdot \left(\frac{sh\xi}{\xi} - ch\xi\right)\right] \cdot \sin^2\theta \qquad (2a)$$

$$\psi = -\frac{k_1 U}{2} \cdot \left[ E\xi^4 + F\xi^2 + G\xi + \frac{H}{\xi}\right] \cdot \sin^2\theta \qquad (2b)$$

$$\tilde{\psi} = -\frac{kU}{2} \cdot \left[ \frac{I}{\chi} + J\chi^2 + L \cdot \left(\frac{ch\chi}{\chi} - sh\chi\right) + M \cdot \left(\frac{sh\chi}{\chi} - ch\chi\right)\right] \cdot \sin^2\theta \qquad (2c)$$

The hydrodynamic force acting upon the reference permeable sphere is given by

$$f = -4G\mu\pi\sqrt{k_1}\, U \qquad (3)$$

The twelve coefficients $A$ to $M$ are determined from the boundary conditions after lengthy calculations. $G$ is explicitly given in the Appendix as a function of $\xi_1, \tilde{c}, k/k_1$.

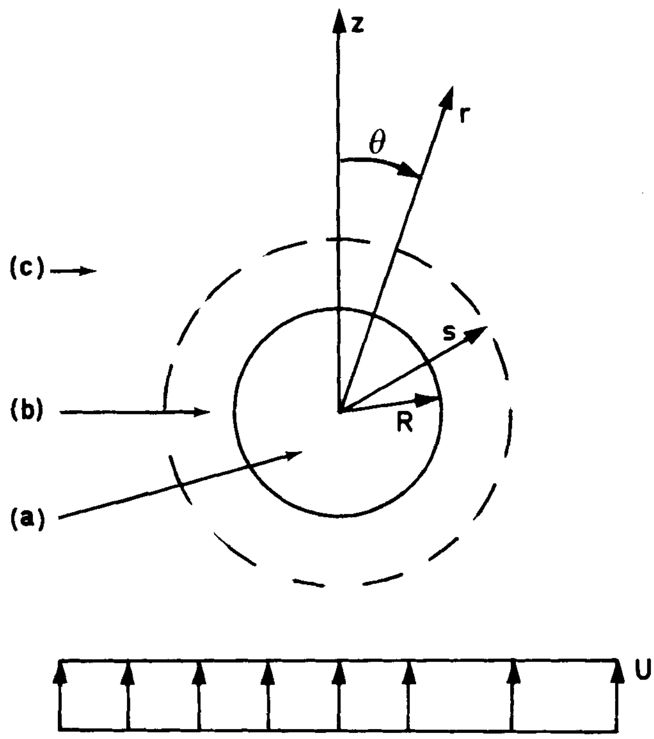


Fig. 1. Description of the modeled system; $R = \tilde{c}^{1/3} \cdot S$. (a). Reference sphere of permeability $k_1$. (b). Fluid region. (c). External porous mass of permeability $k$.

### Closure Equation

This closure equation, necessary for calculating $k/k_1$, is derived by the method of Neale and Nader (1974). When the porous medium is subjected to a uniform pressure gradient, a force $f_1$ is exerted by the fluid on a cylinder of length p and cross-sectional area $s$:

$$f_1 = -\frac{\partial p}{\partial x} \cdot p \cdot s = \frac{\mu U}{k} \cdot p \cdot s \qquad (4)$$

The cylinder contains $N$ permeable spheres:

$$N = \frac{3}{4\pi} \cdot \tilde{c} \cdot \frac{p \cdot s}{R^3} \qquad (5)$$

Thus, the force exerted by the fluid upon each sphere is derived from (4) and (5) as

$$f_2 = \frac{4\pi}{3} \cdot R^3 \cdot \frac{\mu U}{k\tilde{c}} \qquad (6)$$

If we equate $f$ to $f_2$, the closure equation is derived:

$$\frac{k}{k_1} = -\frac{\xi_1^3}{3\tilde{c}\, G(\xi_1, \tilde{c}, k/k_1)} \qquad (7)$$

$k/k_1$ can thus be numerically extracted from this implicit equation as a function of $\xi_1$ and $\tilde{c}$. When $k/k_1$ is known, the two main applications of this formula, that is, permeability and gravitational settling of a swarm of permeable spheres, are straightforward.

## RESULTS AND DISCUSSION

Three limiting cases of (7) were analytically examined. When $\tilde{c} \to 0$, the solution for an isolated permeable

Table 1. Ratio of the Permeabilities $k/k_1$ as a Function of $\tilde{c}$ and $\xi_1$ (Self-Consistent Model)

| $\tilde{c}$ \ $\xi_1$ | 5 | 10 | 20 | 40 | 80 | 160 |
|---|---|---|---|---|---|---|
| 0.10 | 34.79 | 98.27 | 332.4 | 1 228 | 4 731 | 18 580 |
| 0.20 | 13.16 | 32.45 | 100.7 | 356.1 | 1 342 | 5 215 |
| 0.30 | 7.052 | 15.03 | 41.78 | 138.8 | 507.2 | 1 942 |
| 0.40 | 4.411 | 8.045 | 19.29 | 58.30 | 203.1 | 760.6 |
| 0.50 | 3.032 | 4.722 | 9.382 | 24.44 | 78.47 | 282.9 |
| 0.60 | 2.229 | 3.001 | 4.811 | 10.04 | 27.81 | 93.43 |
| 0.70 | 1.726 | 2.061 | 2.700 | 4.257 | 9.097 | 26.30 |
| 0.80 | 1.393 | 1.524 | 1.718 | 2.094 | 3.092 | 6.374 |
| 0.90 | 1.164 | 1.202 | 1.245 | 1.303 | 1.419 | 1.734 |
| 1 | 1 | 1 | 1 | 1 | | 1 |

sphere was obtained (this solution is presented in its correct form by Neale et al., 1973). When $k_1 \to 0$, the implicit equation for the permeability of a swarm of impermeable spheres (Neale and Nader, 1974) was derived. Finally, when $c$ is equated to 1, $k/k_1 = 1$ is the solution of (7), as expected.

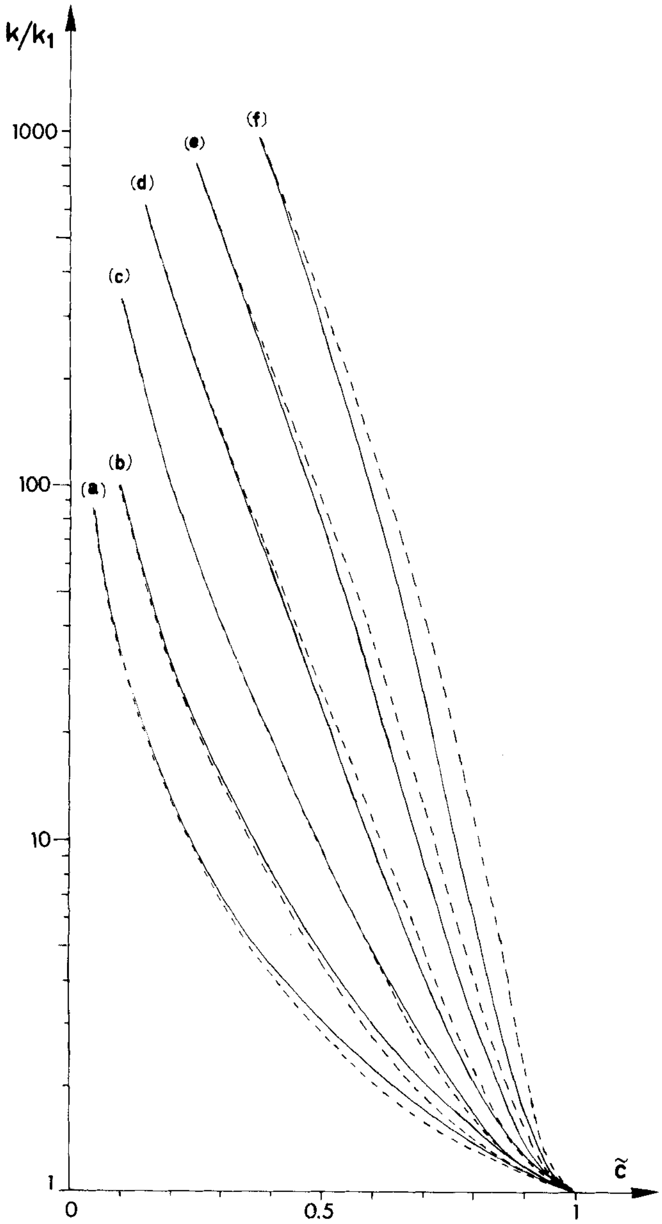


Fig. 2. Ratio of the permeabilities $k/k_1$ as a function of $\tilde{c}$ and $\xi_1$. Broken lines: free cell model. Solid lines: self-consistent model. Values of $\xi_1$ are 5, (a); 10, (b); 20, (c); 40, (d); 80, (e); 160, (f).

Numerical values of $k/k_1$ obtained from (7) are given in Table 1. $k/k_1$ is plotted as a function of $\tilde{c}$ and $\xi_1$ in Figure 2 and compared to the results of the free cell model for permeable spheres (Neale et al., 1973).

For $\tilde{c} \to 0$ and $\tilde{c} = 1$, the results given by the two models are identical. They differ for intermediate values of $\tilde{c}$, and the deviation depends on the value of $\xi_1$; however, the order of magnitude is essentially the same. But some restrictions should be made. For an actual swarm of monosized permeable spheres, the maximum attainable value of $\tilde{c}$ corresponding to a regular geometric packing is 0.74; when spheres are randomly arranged, $\tilde{c}$ is generally lower than 0.65. Moreover, in the case of gravitational settling, $\tilde{c}$ is very rarely greater than 0.4; within this range the results of the two models are virtually indistinguishable, practically speaking (Figure 2).

Finally, in the limiting case where $k_1 = 0$, a comparison between the two models can be found in Neale and Nader (1974).

## CONCLUSIONS

The self-consistent method of Neale and Nader (1974) was applied to the calculation of the permeability of a swarm of permeable spheres. An implicit formula giving the ratio of the permeabilities of the medium and of the spheres was obtained. This formula is more general in character than those given in the literature, since they can be deduced as limiting cases.

Finally, as shown by Neale and Nader (1974), the self-consistent method applied to impermeable spheres leads to good agreement with experimental results. Thus, it could be expected that the same conclusion would hold for the present calculations when permeable spheres are used.

## NOTATION

$A$ to $M$ = coefficients defined by (2); $G$ is detailed in the Appendix

$\tilde{c}$ = volume concentration of permeable spheres

$k_1, k$ = permeability of the spheres, of the whole medium

$p$ = pressure

$r$ = radial coordinate (Figure 1)

$R, S$ = radius of the reference particle, of the cell

$u$ = velocity of flowing fluid

$U$ = mainstream velocity

$z$ = axial coordinate (Figure 1)

### Greek Letters

$\mu$ = viscosity of fluid

$\psi$ = stream function

$\xi, \chi$ = normalized radial coordinates (equal to $r/\sqrt{k_1}$, $r/\sqrt{k}$, respectively)

$\xi_1$ = $R/\sqrt{k_1}$

$\theta$ = polar direction

**Superscripts**

$\hat{}$, $\tilde{}$ = inner permeable sphere, the outer porous medium

The absence of a superscript denotes the fluid region in the cell.

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### APPENDIX

The following formulas give $G$:

$$\alpha = [1/\xi_1 - \text{th}\xi_1(1 + 1/\xi_1^2)] \cdot [\text{th}\xi_1/\xi_1 - 1]^{-1}$$

$$\beta = (k_1/k)^{1/2} \cdot (\chi_2^3 + \chi_2^2 + 2\chi_2 + 2) \cdot (\chi_2^2 + \chi_2 + 1)^{-1}\chi_2^{-1}$$

where $\chi_2 = (k_1/k)^{1/2} \cdot \xi_1 \cdot \tilde{c}^{-1/3}$

$$\mathcal{A} = \xi_1^3 - 5\alpha\,\xi_1^2 + 10\,\xi_1; \quad \mathcal{B} = -\xi_1^2 + 2\alpha\,\xi_1 - 4$$

$$\mathcal{G} = 2/\xi_1 + 2\alpha/\xi_1^2 + 5/\xi_1^3$$

$$\zeta = \xi_1/\tilde{c}^{1/3}; \quad \overline{A} = 3\,\zeta^2 - 90\,k/k_1 + 5\mathcal{B} + 2\xi_1^2\,\mathcal{A}/\zeta^3$$

$$\overline{B} = -\,\mathcal{G} + 3/\zeta + \mathcal{B}/\zeta^3$$

$$\overline{C} = 12\,\zeta^3\,\beta + 30\zeta^2 + 10\zeta\beta(6\,k/k_1 + \mathcal{B}) + 60\,k/k_1 - 2\beta\,\xi_1^2\,\mathcal{A}/\zeta^2$$

$$\overline{D} = 2\zeta\beta\mathcal{G} - 3\beta + 6/\zeta + (\mathcal{B} - 6\,k/k_1)\beta/\zeta^2 + 12\zeta^{-3}k/k_1$$

$$G = 9\overline{C}\cdot(\overline{A}\,\overline{D} + \overline{B}\,\overline{C})^{-1}$$


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# Flow Patterns and Some Holdup Experimental Data in Trickle-Bed Reactors for Foaming, Nonfoaming, and Viscous Organic Liquids


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Determination of flow patterns and liquid holdup in trickle beds is of importance in the modeling, design, and scaling-up of these reactors extensively employed in the hydroprocessing of petroleum fractions (Henry and Gilbert, 1973; Mears, 1974; Paraskos et al., 1975; Satterfield, 1975; Charpentier, 1976). For porous catalyst particles usually employed in trickle-bed reactors, several holdup data have been proposed recently, either for air-water systems (Sato et al., 1973; Goto and Smith, 1975; Colombo et al., 1976) or for gas foaming and nonfoaming hydrocarbons (Satterfield and Way, 1972; Charpentier and Favier, 1975; Midoux et al., 1976; Schwartz et al., 1976). The purpose of this note is to present complementary results on flow patterns and liquid holdup for foaming and nonfoaming organic liquids, to quantify the effect of the liquid viscosity, and to compare these results with those proposed in the literature.

Experiments were carried out in a 5 cm ID column packed to a length of 1.20 m with spherical porous cobalt/molybdenum/aluminum oxide catalyst particle ($d$ = 2.4 mm; $\epsilon$ = 0.385) operating at atmospheric pressure over the range 18° to 25°C. The details of the experimental equipment and procedure to measure the holdup by the weighting method are identical with the ones described by Charpentier and Favier (1975).

Hydrocarbon and organic liquid properties are presented in Table 1. In the presence of a sufficient air flow rate, methanol, kerosene, and desulfurized gas oils have a tendency to foam which does not happen with cyclohexane and ethyleneglycol. As explained previously by Charpentier and Favier, it was not possible to characterize this phenomenon by the physicochemical parameters. Superficial mass velocity of 0.5 to 9 and 0 to 0.9 kg/m$^2$·s for the liquid phases and for the air, respectively, were mainly used to explore the whole field covered by the various flow patterns.

### FLOW PATTERNS

Charpentier et al. (1975, 1976) observed various flow patterns in trickle beds. At low liquid and gas flow rates ($L < 5$ and $G < 0.01$ kg/m$^2$·s), a trickling flow exists where the flow of the liquid is little affected by the flow of the gas (small gas-liquid interaction regime). An increase of gas and/or liquid flow rates leads to pulsing and spray flow for nonfoaming liquids, and foaming, foaming-